

Bound states in the LFD Yukawa model

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Abstract

Our purpose is to calculate relativistic bound states in a quantum field theoretical approach. We work in the Yukawa model and first calculate the bound-state equation in the ladder approximation. We discuss why this is not a complete treatment and what possibilities there are to extend this equation.

1. Introduction

Light Front Dynamics (LFD) is an approach which is very well suited to the problem of calculating bound states in a quantum field theoretical approach (see [1], [2]). In previous work [3] we have calculated the bound states in the LFD approach using a scalar model and compared them with calculations done in the instant form. Here we move to a more realistic model where spin is included for the constituent particles. Work on this subject has already been done by others, see e.g. [4, 5, 6, 7]. One aspect that has not gotten that much attention in the literature is the inclusion of instantaneous terms in a bound-state equation.

In section 2 we detail the model and derive the bound-state equation in the ladder approximation. Section 3 deals with some differences between propagating and instantaneous terms in a bound-state equation. In the last section we give some methods that may be useful in generating an equation which contains the instantaneous as well as propagating terms to all orders.

2. Ladder approximation in the Yukawa model

The model we are using is the Yukawa model, i.e., we have two particles of spin $s = 1/2$ and equal mass m which exchange a scalar particle of mass μ . The Lagrangian for this system is given by:

$$\mathcal{L} = \frac{i}{2} \bar{\Psi} \gamma_\mu (\partial^\mu \Psi) - \frac{i}{2} (\partial^\mu \bar{\Psi}) \gamma_\mu \Psi - m \bar{\Psi} \Psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\mu^2}{2} \phi^2 - g \phi \bar{\Psi} \Psi, \quad (1)$$

where g is the coupling constant, ϕ the scalar field of the exchanged particle and Ψ the field of the constituent particles. Restricting the Fock space to the two- and three-particle sector only and not including any self energy terms, it is possible to derive the bound-state equation from this Lagrangian. Before deriving this equation one has to realize that spin-1/2 constituents have some peculiarities, which result in the fact that we need to express our equation in terms of the independent degrees of freedom: Ψ_+ , $\bar{\Psi}_+$ and ϕ . The free field expansion for the fermionic field, Ψ_+ , involve the 'good' spinors $u_+(p, s)$ (or $v_+(p, s)$). These 'good' spinors can be related to the more commonly used spinors $u(p, s)$ (or $v(p, s)$) via the projection operator Λ_+ ; $u_+(p, s) = \Lambda_+ u(p, s)$. The spinors we use were defined before by Kogut and Soper [8].

Along these lines we find the equation for the bound state of a fermion and an anti-fermion

$$\left[M^2 - \frac{\vec{p}_\perp^2 + m^2}{x(1-x)} \right] \psi(\vec{p}_\perp, x) = \frac{g^2}{2(2\pi)^3} \sum_{\sigma_1, \sigma_2} \int [d^3 p'] \mathcal{K}(\vec{p}_\perp, x; \vec{p}'_\perp, x') \psi(\vec{p}'_\perp, x'). \quad (2)$$

Here is M the total mass of the system, $\mathcal{K}(\vec{p}_\perp, x; \vec{p}'_\perp, x')$ is the kernel of the equation and the integration element is $[d^3 p] = d^2 \vec{p}_\perp dx$. The expression for the kernel is

$$\mathcal{K}(\vec{p}_\perp, x; \vec{p}'_\perp, x') = \frac{1}{\Phi(x, x')} \left(\frac{\theta(x - x')}{2(x - x') D_a} \langle s_1 \sigma_1 | s_2 \sigma_2 \rangle + \frac{\theta(x' - x)}{2(x' - x) D_b} \langle s_1 \sigma_1 | s_2 \sigma_2 \rangle \right), \quad (3)$$

where the phase-space factor $\Phi(x, x')$ is given by

$$\Phi(x, x') = \sqrt{x(1-x)x'(1-x')}, \quad (4)$$

D_a and D_b are the energy denominators corresponding to the two time-orderings

$$\begin{aligned} D_a &= M^2 - \frac{\vec{p}_\perp'^2 + m^2}{x'} - \frac{\vec{p}_\perp^2 + m^2}{1-x} - \frac{(\vec{p}_\perp - \vec{p}_\perp')^2 + \mu^2}{x-x'}, \\ D_b &= M^2 - \frac{\vec{p}_\perp^2 + m^2}{x} - \frac{\vec{p}_\perp'^2 + m^2}{1-x'} - \frac{(\vec{p}_\perp' - \vec{p}_\perp)^2 + \mu^2}{x'-x}, \end{aligned} \quad (5)$$

and the spin matrix elements are given by

$$\langle s_1 \sigma_1 | s_2 \sigma_2 \rangle = \bar{u}(\vec{p}_\perp', x', \sigma_1) u(\vec{p}_\perp, x, s_1) \bar{v}(-\vec{p}_\perp', 1-x', \sigma_2) v(-\vec{p}_\perp, 1-x, s_2). \quad (6)$$

The diagrams corresponding to this bound-state equation are shown in Fig. 1.

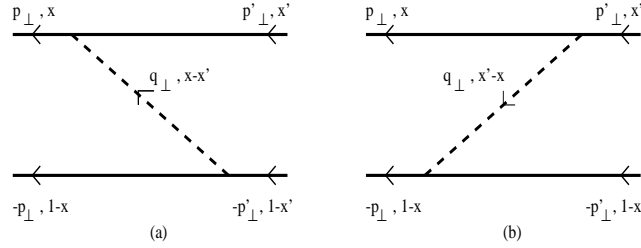


Fig. 1: Time-ordered diagrams in the ladder approximation for the one-particle exchange.

One has to realize that only the propagating diagrams are included in the ladder approximation. Instantaneous terms are not included and we have to add them to our equation to get the complete kernel. The fact that these instantaneous terms are neglected makes it impossible to relate the bound-state equation, Eq. (2), to the Bethe-Salpeter equation, since in the latter one the instantaneous terms are taken into account. In perturbation theory the instantaneous terms were included by e.g. Ligterink and Bakker [9] and in the bound-state equation they were included by Sales et al.[7].

3. Propagating and instantaneous terms

As is shown in [9] it is possible to split the full propagator as follows,

$$\frac{\not{p} + m}{p^2 - m^2 + i\varepsilon} = \sum_{\sigma} \frac{u(p, \sigma) \otimes \bar{u}(p, \sigma)}{p^2 - m^2 + i\varepsilon} + \frac{\gamma^+}{2p^+}, \quad (7)$$

where the first term is the on-shell, propagating part and the second term is the instantaneous term.

As an example to illustrate the differences between the propagating and instantaneous part of the propagator, we will look at one of the time-ordered box diagrams shown in Fig. 2. The diagram on the left contains only propagating terms, the diagram on the right contains also an instantaneous interaction. The amplitudes for these diagrams are denoted by

$$\frac{g^4}{(2\pi)^6} \int [d^3 p''] \mathcal{K}_2^\alpha(\vec{p}_\perp, x; \vec{p}_\perp', x'; \vec{p}_\perp'', x'') \psi(\vec{p}_\perp', x'), \quad (8)$$

where $\alpha = P$ or I , denoting whether we are dealing with the propagating or instantaneous interaction. The kernels in both cases are given by

$$\begin{aligned} \mathcal{K}_2^P &= \frac{\theta(x'' - x)\theta(x'' - x')}{\Phi^P D_a D_b D_c} \bar{u}(\vec{p}_\perp', x', \sigma_1) \Lambda u(\vec{p}_\perp, x, s_1) \bar{v}(-\vec{p}_\perp', 1-x', \sigma_2) \Lambda v(-\vec{p}_\perp, 1-x, s_2), \\ \mathcal{K}_2^I &= \frac{\theta(x'' - x)\theta(x'' - x')}{\Phi^I D_a D_c} \bar{u}(\vec{p}_\perp', x', \sigma_1) \gamma^+ u(\vec{p}_\perp, x, s_1) \bar{v}(-\vec{p}_\perp', 1-x', \sigma_2) \Lambda v(-\vec{p}_\perp, 1-x, s_2) \end{aligned} \quad (9)$$

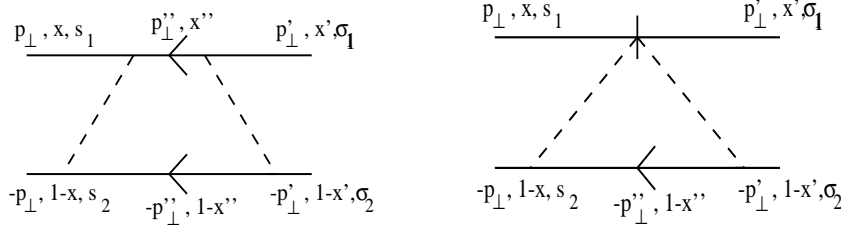


Fig. 2: Two time-ordered diagrams of the covariant box diagram, one containing only propagating terms (left) and one containing also an instantaneous terms (right).

where $\Lambda = \sum_{\sigma} u(p, \sigma) \otimes \bar{u}(p, \sigma)$ and corresponds to the propagating fermion and γ^+ to the instantaneous propagator. The energy denominators D_i are given by

$$\begin{aligned} D_a &= M^2 - \frac{\vec{p}_{\perp}^2 + m^2}{x} - \frac{\vec{p}_{\perp}''^2 + m^2}{1 - x''} - \frac{(\vec{p}_{\perp}'' - \vec{p}_{\perp})^2 + \mu^2}{x'' - x}, \\ D_b &= M^2 - \frac{\vec{p}_{\perp}''^2 + m^2}{x''} - \frac{\vec{p}_{\perp}^2 + m^2}{1 - x''}, \\ D_c &= M^2 - \frac{\vec{p}_{\perp}'^2 + m^2}{x'} - \frac{\vec{p}_{\perp}''^2 + m^2}{1 - x''} - \frac{(\vec{p}_{\perp}'' - \vec{p}_{\perp}')^2 + \mu^2}{x'' - x'}, \end{aligned} \quad (10)$$

and the phase-space factors Φ^{α} are given by

$$\begin{aligned} \Phi^P &= 4\sqrt{x(1-x)x'(1-x')}x''(1-x'')(x''-x')(x''-x), \\ \Phi^I &= 4\sqrt{x(1-x)x'(1-x')}(1-x'')(x''-x')(x''-x). \end{aligned} \quad (11)$$

The differences in the expressions for these diagrams lie in the definition of the phase-space factor, the number of energy denominators which are present in the kernel and of course the spin matrix elements. Looking at the expressions for both diagrams, we see that they both have an ultraviolet logarithmic divergence.

In [9] it was shown that in order to get agreement between a covariant and a LFD calculation one needs to add diagrams containing an instantaneous fermion to ones that contain only propagating fermions. This way a *blink* is constructed and the divergences of both diagrams appear to cancel. The two diagrams in Fig. 2 can be added this way, constructing a *blink* on the upper line.

Using this *blink* mechanism, we are able to include the instantaneous diagrams in the bound-state equation. Unfortunately, we have to add them by hand. Which means that we will get an equation which is correct up to order g^2, g^4 , etc. Using this formalism, it seems impossible to generate an equation in such a way that we include the instantaneous terms and get the equation correct to all orders.

4. Other methods

One possibility to generate such an equation is given in [7]. Here the light-front equation is split into two equations

$$\begin{aligned} T(K) &= W(K) + W(K)\tilde{G}_0(K)T(K) \\ W(K) &= V(K) + V(K)\left[G_0^F(K) - \tilde{G}_0(K)\right]W(K), \end{aligned} \quad (12)$$

which generate the bound-state equation. Here $T(K)$ is the transition matrix, $V(K)$ and $W(K)$ are driving terms and $G_0^F(K)$ and $\tilde{G}_0(K)$ are Green's functions. The question is whether one can generate

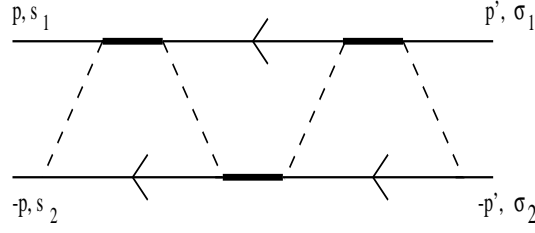


Fig. 3: Higher order diagram with *blinks* on both lines.

all diagrams with these two equations. We have to look whether it is possible to generate for example a diagram with *blinks* on both legs as is shown in Fig. 3.

An other possible way to generate an equation which has the instantaneous terms included to all orders, was introduced by Pang and Ji [10]. The full propagator is split as follows

$$\frac{\not{p} + m}{p^2 - m^2 + i\varepsilon} = \sum_{\lambda} \frac{u_{\lambda}(p)\bar{u}_{\lambda}(p)}{p^2 - m^2 + i\varepsilon} + \frac{w_{\lambda}(p)\bar{w}_{\lambda}(p)}{p^2 - m^2 + i\varepsilon}, \quad (13)$$

where the w -spinors are given by (note that the representation used in [10] is different from the one we use)

$$w_{\uparrow}(p) = \frac{\varepsilon(p)}{\sqrt{2p^+}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad w_{\downarrow}(p) = \frac{\varepsilon(p)}{\sqrt{2p^+}} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad (14)$$

where $\varepsilon^2(p) = p^2 - m^2$. Further investigation is needed to see whether it is possible to generate an equation which has all propagating and instantaneous terms included.

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